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Computational Needs for Muon Accelerators

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Muon accelerators contain beam lines and components which are unlike any found in existing accelerators. Production of the muons requires targets for beams with powers which are at or beyond what has currently been achieved. Many subsystems use solenoid focusing systems where at any given point, several magnets have a significant influence. The beams that are transported can have energy spreads of ±30% or more. The required emittances necessitate accurate tracking of particles with angles of tenths of a radian and which are positioned almost at the edge of the beam pipe. Tracking must be done not only in vacuum, but also in materials; therefore, statistical fluctuations must also be included.

Design and simulation of muon accelerators requires software which can: accurately simulate the dynamics of solid and liquid targets under proton bombardment; predict the production of particles from these targets; accurately compute magnetic fields based on either a real magnet design or a model which includes end fields; and accurately design and simulate a beam line where the transported beam satisfies the above specifications and the beam line contains non-standard, overlapping elements. The requirements for computational tools will be discussed, the capabilities of existing tools will be described and compared to what is required.

1. Introduction

In recent years, there has been an increased interest in machines which transport and accelerate muons instead of the more traditional electrons or protons [1], especially in light of the recent experimental results in neutrino physics [2–8].

Unlike the more commonly-used electrons and protons, muons decay in a relatively short amount of time, and this places strong requirements on any system that is transporting or accelerating them. Furthermore, the method of muon production, bombarding a target with protons and capturing the muons that result from the decay of the produced pions, leads to the muons being produced in a very large phase space volume. As a result, the muon beams will almost completely fill the beam pipe, while at the same time particles in the beam will make angles with respect to the reference orbit of as much as 0.3 rad. (especially in the ionization cooling section). There may be an energy range of a factor of two within a given beam. Furthermore, for acceleration, there has recently been interest in using FFAGs [9]. These are circular machines with a beamline that accepts a factor of two or more in energy. These are conditions which are not often encountered in traditional particle accelerators, and therefore must be handled very carefully.

The large number of muons that must be produced requires that a large number of protons hit the target. For several reasons, this generally should be done in a small number of high-energy pulses, rather than a large number of low-energy pulses or even continuously. The RF power systems for acceleration and cooling require too much power and/or cooling to be run continuously, so they are generally run in a pulsed mode; the average power used in these systems will be proportional to their repetition rate. For collider applications, fewer pulses will lead to a higher luminosity. For neutrino applications where the detector is not buried deep underground, background elimination will be more effective with a lower pulse rate. The high average power and pulse energy on the target will often damage or destroy the target; modeling and predicting this is important to muon accelerator design.

Finally, the requirements of rapid acceleration and cooling as well as large longitudinal phase space acceptance require high RF gradients. The

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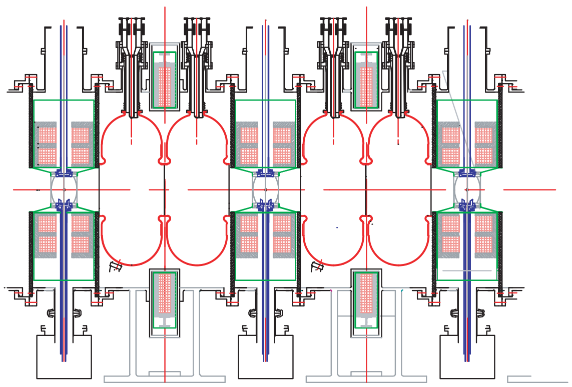

Figure 1. Example of two cells of a cooling lattice.

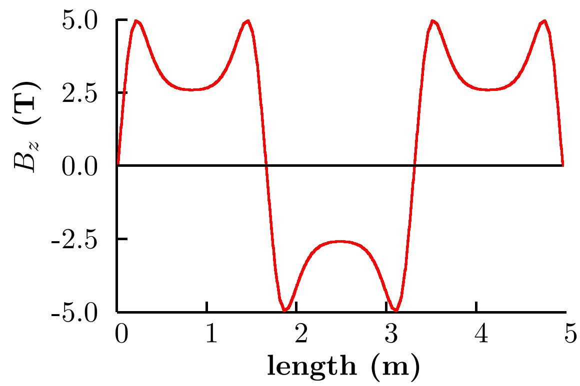

Figure 2. Longitudinal field as a function of position in a cooling cell.

gradient which can be achieved in cavities is limited by RF breakdown. To complicate matters, these cavities are in a high magnetic field, which has a strong effect on the breakdown [10]. This phenomenon is not well-understood at this point, making predictive computations difficult. Hence, this topic will not be addressed here in detail. Understanding and predicting that process is important for the design of cavities for muon cooling and for the prediction of achievable RF gradients.

2. Beam Dynamics

A muon beam transport line must deal with beams having an especially large phase space area. In addition, many beamlines must accommodate a beam whose central energy and position will vary over time as the beam is accelerated (specifically, FFAGs). Many accelerator design and tracking codes, often without stating so, implicitly assume that at least some dimensions of the beam's phase space are small. Any code which is used in analyzing muon accelerators must be cautious to include all dynamical effects.

2.1. Magnetic Fields

To transport a large-emittance beam, it is necessary to have a highly compact lattice. Figure 1 shows an example of the lattice for a cooling cell. There are solenoids around the cavities and at the ends of the cavities. The longitudinal field as a function of position in the cell is shown in Fig. 2. It can be seen from this field that the solenoids are not well-approximated by constant-field separated magnets. In fact, the variation of the field with position leads to significant nonlinearities, the correct modeling of which is critical for understanding the performance of the lattice.

For another example, consider the RFOFO cooling ring shown in Fig. 3. This ring consists exclusively of solenoids, but the solenoids are tilted so as to produce a bending field. The horizontal and vertical fields on the circle shown in Fig. 3 are shown in Fig. 4. As discussed before, this field cannot be computed by looking at one magnet at a time, and the longitudinal variation of the field will produce significant nonlinearities which must be accurately computed. But this further illustrates a problem of coordinate system representation. The optimal way to represent the particle positions for this ring is most likely by deviations from the circle shown in Fig. 3. However, many codes would instead use a coordinate system based on a path that curves as a particle at a "reference energy" moving in the magnetic would. As can be seen from Fig. 4, this would lead to a trajectory that has non-uniform curvature, and bends in the vertical plane as well. Ensuring that the resulting reference curve bends

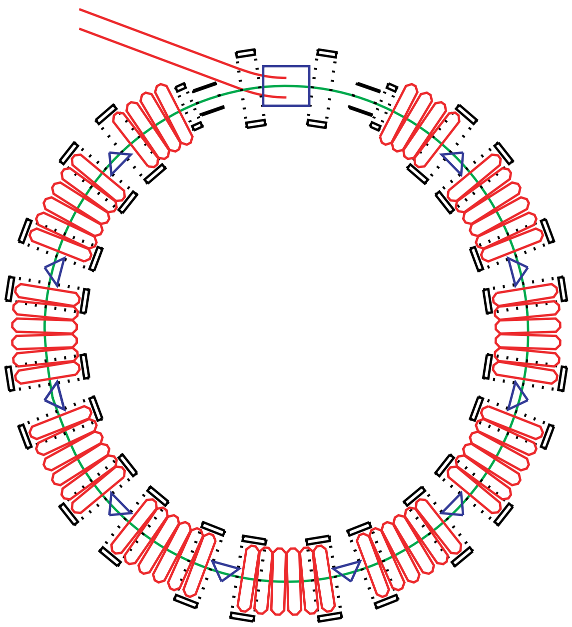

Figure 3. Layout of a Cooling Ring

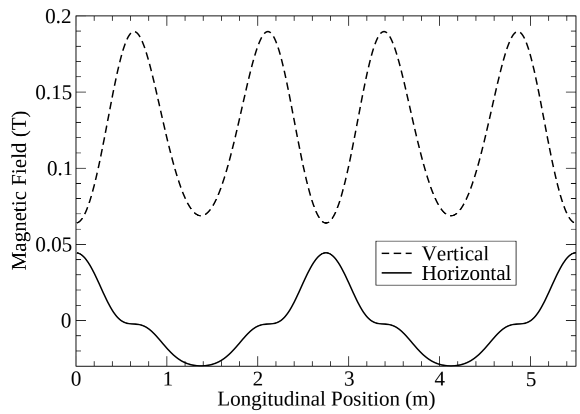

Figure 4. Horizontal and vertical fields in a cooling ring.

by the appropriate amount per cell and doesn't have a vertical displacement from beginning to end would be very challenging, and would in fact require one to start out working in some coordinate system that is independent of the magnetic field in the first place. Thus, a code for complex systems such as this must be capable of working in a coordinate system that is not determined by the magnet fields. Since in this case, no particle actually follows the reference curve (but should stay near it), one must be sure to handle RF synchronization properly, presumably by making a preliminary pass to find a closed orbit.

Many nonlinear effects result not from intentionally introduced nonlinearities, but from nonlinearities that are the result (due to Maxwell's equations) of the longitudinal variation of linear fields (Figs. 2 and 4). Any tracking or analysis code must be able to describe the fields that a particle sees in a way that is consistent (at least to some level of accuracy) with Maxwell's equations. While this can often be done with a field map, a field map for a complex lattice can be extremely large, and will often require an external magnet design code to compute. Thus, the capability of finding a field from more compact information (such as the longitudinal variation of multipole components) is an extremely useful capability of any tracking or analysis code for muon accelerators.

2.2. Truncated Power Series

Many accelerator design codes use a truncated power series representation as part of their analysis. When a beam has a large phase space area, there is a concern with the rate of convergence for the power series at large amplitudes.

In most cases, a truncated power series will give an adequate description of the transfer map through a short section of the lattice. However, the transfer map for a longer section of lattice may not give an accurate representation of the dynamics. Figure 5 shows the trace of the transfer matrix (which should be between -2 and 2 for stability) for 10 cells (each cells contains a single triplet) of an FFAG lattice. Note that for even a 10th order power series, the trace of the transfer matrix is so inaccurate as to give the incorrect results for the range of stability. It is in fact possible to construct a sufficient anomalous triplet

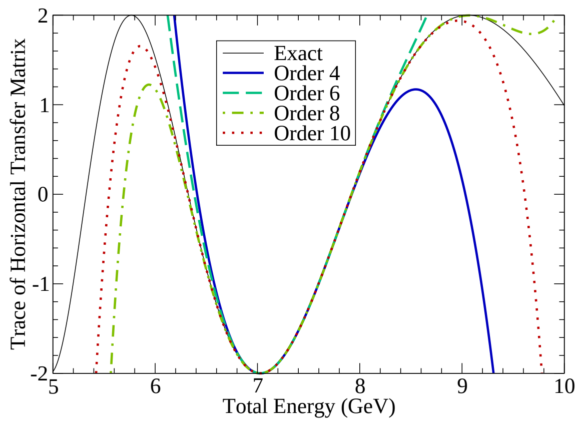

Figure 5. Trace of the horizontal transfer matrix for 10 cells of an FFAG lattice computed using a truncated power series [11].

lattice where the power series fails to converge within the operating range of the lattice.

The basic difficulty is that in composing two truncated power series and then re-truncating, information is lost in the final truncation.

This should not lead to ruling out truncated power series as a useful technique for analyzing these lattices, but they should be used with extreme caution. Maps mad for a long section of lattice are more likely to have problems than those made from shorter sections. Many algorithms based on truncated power series require a map for the entire ring [12]. Some analysis can be performed by making a power series and fixed energy for several different energies. In fact, most FFAG analysis occurs by finding a closed orbits at different energies and computing the linear maps about those closed orbits [13].

2.3. Analysis Techniques

In the design of muon accelerators, many of the questions asked are different than those that are asked in traditional accelerators. This is true especially for cooling: one may want to compute the equilibrium emittance, the rate of cooling, and various related merit factors.

Muon cooling systems have generally been designed using tracking. However, using tracking has many difficulties. Often one is trying to compare some merit factor for two different designs, and the statistical fluctuations due to using a finite number of particles make differences in the merit factor difficult to ascertain. Using more particles would be helpful, but due to the complexity of the computation of the fields as described above, it becomes prohibitive to run large numbers of particles. Thus, improving the efficiency of field computations would allow more effective machine design. Making codes such as ICOOL [14] run on parallel computers would also be useful, and will probably be necessary for any final machine design.

Ideally, we would like to have techniques available for cooling lattice design that do not involve simply tracking particles. Such an analysis code should be able to compute the usual quantities (linear maps, closed orbits), but should also include effects such as cooling and multiple scattering (in some averaged way). Some theoretical work on this subject has been done, but under restricted conditions [15]. This theory has yet to be implemented in any analysis codes.

One may prefer a library which implements accelerator computations instead of a single code; this would allow one to use a higher-level language to compute complex computations on the results of simpler lattice computations.

3. Target

To achieve acceptable physics results, muon accelerators require the production of large quantities of muons. The weak interactions of neutrinos with matter means that to produce a reasonable number of events, one must start with large quantities of muons. For a muon collider, the diffuse phase space in which the muons are produced requires that luminosity be achieved with large numbers of muons instead of very small beams.

In designing the muon production system, the first thing one must know is the number of pions and muons produced. For the design of the subsequent capture systems, one must also know their energy and angular spectrum. One must be able to compute particle production to com-

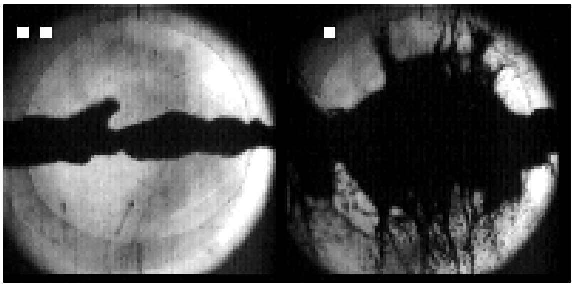

Figure 6. Experimental results showing the mercury jet before (left) and after (right) being hit with a proton beam [26].

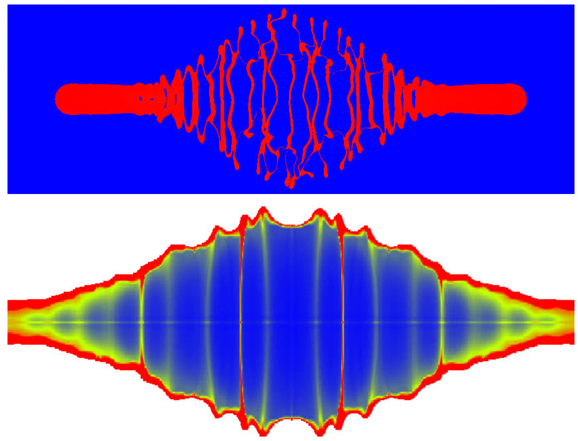

Figure 7. Simulation of interaction between the beam and the liquid mercury jet with cavitation: top is with the two-phase model, bottom is with the homogeneous equation of state.

pute energy deposition in the target area materials so as to design the cooling system. And one must understand how materials will behave under this irradiation, both for the purposes of radiation protection as well as material lifetime and degradation of material properties.

There are several codes which compute particle interactions with matter [16–19]. However, there are uncertainties in the results of those codes by as much as 30% in some regimes [21,22]. These uncertainties may arise both from the algorithms in the codes as well as the unknown nature of the physical processes. If we want to improve prediction accuracy for target design, these uncertainties must be reduced.

Target damage and destruction is a significant problem to be avoided for any high-power target. It is believed that if the energy deposition in and properties of the material are known, existing codes can predict the point at which the target will be damaged [23]. However, the material properties, such as the coefficient of thermal expansion and the yield strength, will change under irradiation [24]. One must be able to predict this change computationally, which at this point requires experimental results to give parameters to the computation.

To avoid the problem of breaking a solid target, a liquid mercury jet has been proposed [25]. The evolution of this jet under proton bombardment and in a magnetic field must be simulated to ascertain whether the jet will be sufficiently stable to be hit by a proton beam, whether earlier portions of the jet will interfere with later portions of the jet or with pion production, and other kinds of effects (see Fig. 6). Parts of the liquid mercury will become vapor as a result of the proton energy deposition and the subsequent pressure waves in the mercury. This "cavitation" process is essential to correctly modeling the mercury jet. Code has been written which models the fluid using a two-phase equation of state [27]. This has been only done in two dimensions; it can in principle be done in three dimensions, but is computationally expensive. Another approach is to treat the fluid as a homogeneous "bubbly fluid" using an equation of state [28]; this has been done in three dimensions. While simulations without cavitation (either model) have been performed without magnetic fields, simulations with cavitation or the bubbly fluid model and magnetic fields have not. Furthermore, as can be deduced from Fig. 6, the evolution of the jet in the nozzle (including magnetic fields) is important for producing a good target, but has not been studied as yet.

4. Conclusion

To design and predict the performance of muon accelerators, one must carefully simulate all aspects of the machine. Single-particle dynamics are well understood, but must be handled with particular care due to the characteristics of the muon beam and the beamline guiding it. Targets must be carefully studied to ensure that they will perform as needed and survive. Understanding of RF breakdown, especially in a magnetic field, must be obtained to attempt to maximize the gradients in room-temperature rf cavities.

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